i-PERCEPTION
a Pion publication
i-Perception (2013) volume 4, pages 532–542
dx.doi.org/10.1068/i0609 ISSN 2041-6695 perceptionweb.com/i-perception

Exocentric pointing in the visual field

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Received 28 June 2013, in revised form 20 August 2013; published online 9 December 2013

Abstract. "Exocentric pointing in the visual field" involves the setting of a pointer so as to visually point to a target, where both pointer and target are objects in the visual field. Phenomenologically, such pointings show systematic deviations from veridicality of several degrees. The errors are very small in the vertical and horizontal directions, but appreciable in oblique directions. The magnitude of the error is largely independent of the distance between pointer and target for stretches in the range 2–27°. A general conclusion is that the visual field cannot be described in terms of one of the classical homogeneous spaces, or, alternatively, that the results from pointing involve mechanisms that come after geometry proper has been established.

Keywords: orientation, direction, anisotropy, geometry, space

1 Introduction

There exists an extensive literature on pointing towards targets under visual control. Much of it involves pointing to locations in three-dimensional space (perhaps considering visuomotor factors), but there is also a large literature on egocentric pointing towards locations in the (two-dimensional) visual field. In this study we address a categorically different type of pointing, which might be called "exocentric pointing in the visual field." We have earlier studied exocentric pointing in the visual world (Koenderink, van Doorn, & Lappin, 2000) and in pictorial space (Wagemans, van Doorn, & Koenderink, 2011).

Exocentric pointing in the visual field occurs indeed fully inside the visual field,[1] which contains both a pointer and a target. The observer is given control over the direction of the pointer, and the task is to let it (visually) point towards the target. The relation with eye position is initially undefined in this case, different from the various forms of egocentric pointing, where eye position tends to be a crucial factor.

We were led to consider this task because we accidentally ran into an unexpected instance of systematic deviations in pointing for the case of an experiment on pictorial space (Wagemans et al., 2011). Pictorial space is a three-dimensional—or perhaps two-plus-one-dimensional (Koenderink, van Doorn, & Wagemans, 2011)—manifold, thus different from the visual field. In the latter experiment the observer had to direct a pointer towards a target in pictorial space. In this paradigm the spatial attitude of the pointer has two degrees of freedom, namely a slant involving the depth dimension, and a tilt that is defined in the visual field. We were primarily interested in the slant, and fully expected the tilt component to be trivial—thus not informative, a component to be discarded in the subsequent analysis. To our surprise we detected a pattern of systematic tilt pointing errors that repeated consistently over sessions and participants. This led us to embark on the present study.

[1]A definition of the "visual field" (Smythies, 1996) is: The visual field is the *spatial array of visual sensations available to observation in introspectionist psychological experiments*, while "field of view" *refers to the physical objects and light sources in the external world that impinge the retina*. Whereas we object to Smythies' notion of field of view, the definition of visual field will do for our paper.

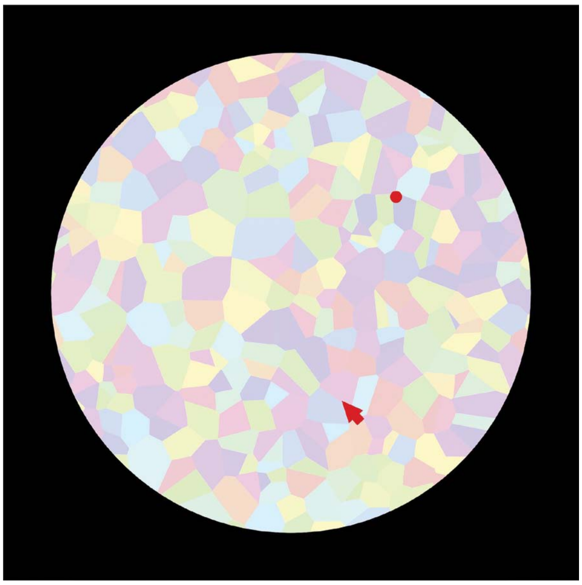

Figure 1. The stimulus array with a target (the red circular disk), and a pointer (the red arrow), superimposed on a circular background disk filled with random Voronoi cells of 20% saturation. The texture was refreshed on each trial, in order to prevent the observer from using landmarks. In this figure (as in Figure 10), we made no attempt to reproduce the anti-aliasing used in the actual display. Pointer and target appear at random locations, the pointer in some random direction (in the example apparently quite far "off").

In this study we present the observer with a bright circular area of fairly large diameter in an otherwise dark surround (Figure 1). This should minimize cues as to the absolute vertical or horizontal, or certain oblique directions (as would be the case with the diagonals of a rectangular area). The field is filled with a statistically uniform and isotropic low-contrast texture. This serves to visually define the fronto-parallel plane, as a uniform luminance would not. (For instance, in a Ganzfeld spatiality is virtually indeterminate.) Then we generate random pairs of locations within the area. At one location we draw a pointer, at the other a target. The observer is given control over the direction of the pointer. The task is to let the pointer visually point to the target. This is repeated for a great many pairs. This particular stimulus arrangement closely mimics the aforementioned case of pointing in pictorial space. However, it is a purely two-dimensional task,[1] involving no depth at all.

Perhaps surprisingly, we found no published data on this particular task. A reason might be that there appear to be no obvious applications of it in real-life settings. However, there exists an extensive literature on the discrimination and matching of directions in the visual field that may perhaps bear on the present case. We discuss such possible relations in the final section of this paper.

2 Methods

2.1 Participants

In Wagemans et al. (2011) and subsequent experiments we detected the two-dimensional tilt effect in the results of about a dozen observers, without a single exception. Thus we decided that just a few observers would amply suffice in the present task. They are the authors (AD female, JK and JW male), for which we also have extensive data on the pictorial space (two visual field dimensions augmented with a depth dimension) paradigm. All have normal, or corrected-to-normal vision. Observers used their preferred eye, right for AD and JW, left for JK. (This has possible implications for the results, as discussed later.) For none of the observers is eye dominance especially marked. Head position was controlled by means of a chin rest.

2.2 Experimental procedure

No particular fixation instructions were given. Introspectively, the participants looked back and forth between pointer and target locations, but fixation locations were not monitored.

The direction of the pointer could be controlled by the observer in hardly discriminable steps through two buttons (a clockwise and a counter-clockwise one). The step-size was more than an order of magnitude smaller than the standard deviations in repeated settings. A third button could be pressed so as to increase the step-size 10-fold, enabling faster approach to the intended direction.

The experiment was self-paced. When the participant indicated satisfaction with a setting (by hitting a button), the next pointer–target pair immediately appeared. The participants performed a thousand settings in a period of about an hour to an hour and a half. Thus they spent a few seconds on each setting.

2.3 Stimulus configuration

The stimuli were presented on a DELL U2410f monitor and a 1,920 × 1,200 pixels liquid crystal display (LCD) screen, in a darkened room. The luminance of the stimulus area was 130 cd/m^2, while that of the "black" surrounding screen area was 0.8 cd/m^2. The luminance of the walls of the room was more than a factor 1,000 lower than the stimulus luminance. The viewing distance was 78 cm. The circular area had a diameter of 24.2º of visual angle. It was filled by a low contrast (20% saturation) random Voronoi cells texture (Figure 1).

Pointer and target were colored red, so as to immediately draw attention. The pointer had a length of 88.5′, width of 29.5′ of visual angle. The target was a circular disk with a diameter of 36.9′ of visual angle. Special efforts were devoted to minimize possible artifacts due to the pixelation. The monitor has a pixel spacing of 1.54′ of visual angle. We used high-quality anti-aliasing, and avoided a right angle at the tip in the design of the pointer. We are confident that pixelation artifacts can be ignored in the interpretation of the results.

The pointer was rotatable about its fiducial location. The locations of both pointer and target were drawn at random from a uniform distribution, subject to the conditions that the pointer should in no attitude touch either the target or the boundary of the background disk. Thus the distance between target and pointer (henceforth denoted "stretch") varies from about 2º to 27º. This is considerably larger than typical sizes used in discrimination and matching studies. Another (related) difference is that we used no fixation mark.

For an additional experiment the pointer length was increased to 19.5º of visual angle, and the pointer width to 1.2º of visual angle. In this experiment there was no target, and the pointer was rotated about the center of the background disk.

2.4 Experimental data

The locations of pointer and target, the final direction set by the observer, and the time taken to arrive at the setting were recorded on file. For the analysis the response times are ignored, and the locations are converted to fiducial direction and mutual separation. For the initial analysis only the fiducial and the responded directions are taken into account.

3 Main experiment: Pointing between a pair of locations

In the main experiment the task is to set the pointer such as to visually point to the target. The stimulus configuration is shown in Figure 1. Participants consider this a simple, even trivial task. It is a monotonous task. Achieving a thousand settings takes some determination. Participants are always fully confident of their settings. The presence of systematic deviations in the settings comes as a surprise to them, after the conclusion of the experiment. Yet such systematic deviations are immediately apparent in a scatterplot of the responses as a function of the fiducial directions (Figure 2).

The deviations are similar for all participants, and neatly repeat the findings from the pictorial space experiment. This corroborates our intuition that these deviations are a pure visual field phenomenon, and have nothing to do with pictorial space per se. It also indicates that the rectangular frame in the pictorial space experiment does not serve to "explain" the effect. This is not a trivial finding. In fact, we had a priori considered it to be the most likely explanation. In order to double-check we repeated the experiment with the monitor rotated over 45º about the viewing direction. This induced the expected 45º phase shift in the settings. Another check involved an internally blackened cylindrical optical tunnel, effectively removing all horizontal–vertical references that might occur in the room or the monitor. It made no difference to the results.

A much better view of the data is obtained in a scatterplot of the deviations from veridical against fiducial direction. One trivially finds a periodic function, since the fiducial direction repeats with a period of 360º. It is not trivial that the deviations systematically conform to a periodicity of 90º (Figure 3). Indeed, the best fitting sine function of that periodicity captures much of the systematic variation. The amplitude of this component is appreciable:

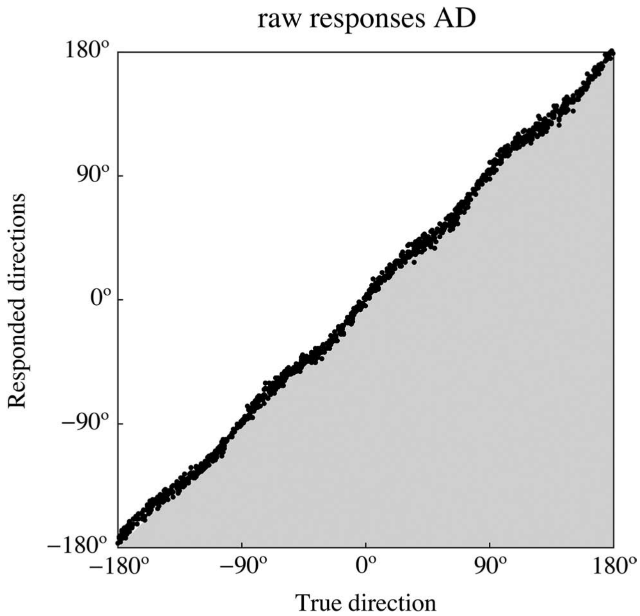

Figure 2. The raw results for participant AD. It is a scatterplot of the responded directions against the fiducial directions. Notice that the points fail to cling to the main diagonal (indicating veridical response, shown as the edge of the gray area), but systematically deviate from it, albeit only slightly. Notice also that the spread is least near fiducial directions that are multiples of 90°. (Horizontal to the right is 0°, vertically upwards 90°.) The curve is steepest at the horizontal and vertical directions, implying that the mis-pointings are towards the diagonal directions.

AD 4.60°

JK 3.21°

JW 3.92°

Notice that this implies a total variation of about 10°. Thus the errors are indeed substantial. They are of the same order of magnitude as those for the matching of directions (for line elements at some mutual separation) as reported by Bouma and Andriessen (1968; see also Wenderoth & White, 1979). Similar effects are found in orientation judgments (Girshick, Landy, & Simoncelli, 2011). Much smaller effects are found in discrimination tasks (Westheimer & Beard, 1998). However, in the present task the observers are not matching a given direction, but instead they directly point to a target. (In fact, there are many crucial differences that render such comparisons rather questionable.)

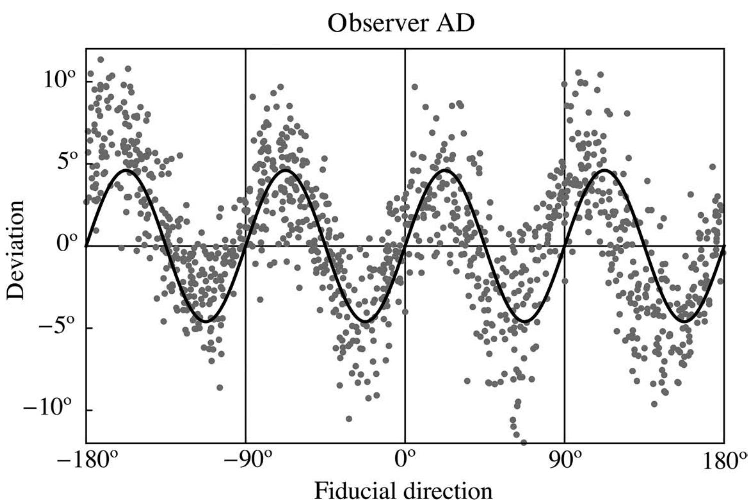

Figure 3. This is essentially the same data as shown in Figure 2, except that we have plotted the deviations from veridicality, rather than the responded directions. The gray points are the data, the black curve is the best fitting sine of period 90°.

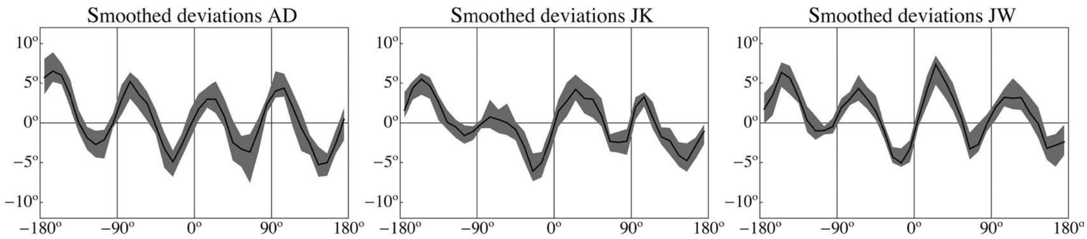

Figure 4. A summary of the data shown in Figure 3 (for AD only), obtained by running a boxcar filter of width 10º over the data. This allows us to show the median trend (black curve), as well as the spread (the running inter-quartile range). Here the data for all three participants is shown, revealing inter-observer variations.

The fit of the sine function with period 90º is evidently not perfect. In order to study the systematic nature of the data more precisely, we ran the data through a boxcar filter of width 10º. This has the effect of smoothing the raw data, yet preserving the systematic structure. The window width is large enough to admit fairly precise estimates of quartile values. In Figure 4, we plot the running median and quartile ranges for all participants. This reveals evident inter-observer differences.

The smoothed data of Figure 4 allow a fairly precise estimate of the zero crossings that are the directions into which the median of the pointings happens to be veridical (Figure 5). Apparently the observers largely agree, and are close to veridical for the horizontal or vertical directions, but somewhat deviate from each other, and have appreciable spread in the diagonal directions.

It seems a priori likely that the deviation will depend on the stretch to be covered by the pointing, that is, the distance between the locations of target and pointer. In order to check this we divided the data in short, medium, and long stretches. Short stretches are defined as stretches shorter than the 25% quartile of all stretches, whereas long stretches are defined as stretches longer than the 75% quartile of all stretches. A comparison of the short and long stretches (this involves only one half of the data, of course) is shown in Figure 6 for participant AD.

The differences can be quantified by fitting 90º period sine waves to the data, and finding the ratio of amplitudes for the long as compared to the short stretches. The results are:

AD 1.33
JK 0.88
JW 1.27

In the case of a generic geometrical structure one expects the amplitudes to be roughly proportional to the stretches (see Figure 7). This is evidently not the case. The conclusion is that the stretch has only a minor influence on the deviations.

This is a surprising conclusion with far-reaching implications. Here is why: The visual field is apparently unlike the Euclidean plane. Can it be described as a "geometry" at all? One would expect so given the fact that this is generally assumed in virtually every textbook, with the exceptions of non-generic cases (e.g., amblyopia or tarachopia; Hess, 1982). Here we might first consider the simplest "geometries," which are the classical homogeneous spaces. These are spaces that appear the same from any point, and that admit of a continuous group of "movements," or "congruences." There are three qualitatively distinct contenders for the (non-degenerate) geometry of the plane (Coxeter, 1961,

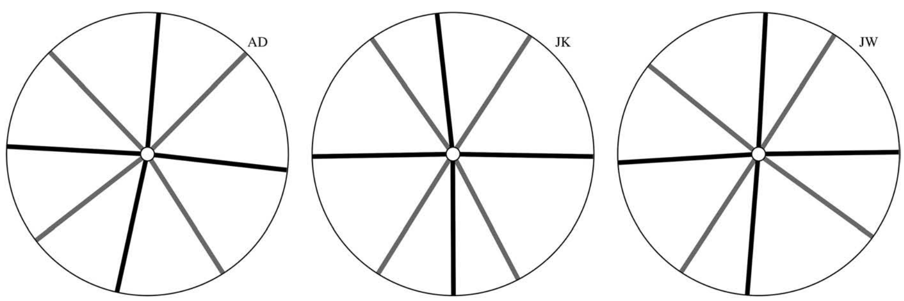

Figure 5. The zero crossings calculated from the smoothed data shown in Figure 4. These plots allow a more intuitive overview of the data. For ease of reference the zero crossings corresponding to the horizontal and vertical directions have been printed black, those corresponding to the oblique directions gray.

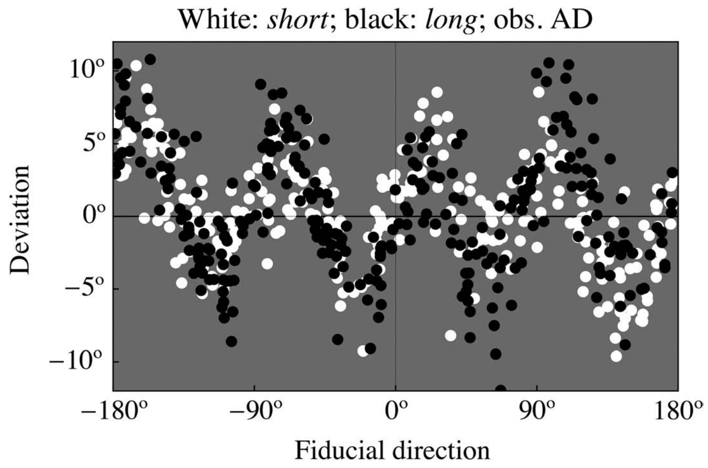

Figure 6. Here the short (white points) and long (black points) stretches have been selected from the raw data. The results for the other participants look very similar.

1965).[2] In each of these the infinitesimal neighborhood of any point is Euclidean; thus a *unique geodesic* passes through any point in any given direction.[3] The Euclidean straight lines through the origin cannot be geodesics of some simple geometry, because then our results reveal the neighborhood of the origin to be non-projective (Coxeter, 1949).[4,5] (See Figure 8.) But when the Euclidean line is not a geodesic itself, each of its points lies on a distinct geodesic (like the configuration in Figure 7), and points at different distances from the origin would be at different directions from the origin. However, this is contradicted by our empirical results. The conclusion is that the visual field cannot be described as one of the classical planar geometries at all. An extreme example of the kind of structure we mean here is illustrated in Figure 9. This is a very general, and perhaps unexpected finding.

Perhaps oddly, the visual field has many properties of a homogeneous plane (close to Euclidean). The mis-pointings appear to be due to some factor that kicks in “after” the geometry, like some add-on specific to the pointing task.

Our task involves eye movements, due to the size of the display area. This might conceivably induce mis-pointings in the diagonal directions (not in the horizontal/vertical directions), as formalized in Helmholtz’s (1867) explanation of subjective curvatures due to Listing’s Law. However, the deviations we meet with are much larger than what the Helmholtz model would predict.

4 Additional experiment: Pointing towards absolute directions

The results of the main experiments give rise to a variety of questions. One type of question is related to the difference between the horizontal/vertical directions, and the diagonal directions. For instance, it has been suggested that the detection of verticality is a categorical judgment, whereas the detection of arbitrary directions is of an analog nature (Quinn, 2004). In order to get at least some grip on these problems we ran an additional experiment.

In this experiment we eliminated the target, and rotated the pointer about the center of the circular background disk (Figure 10). The pointer itself was greatly increased in size. The task of the

[2]These are the Euclidean plane, Lobachevsky’s hyperbolic plane, and Riemann’s elliptic plane (Coxeter, 1961). Our reasoning applies equally to the much more general Riemannian planes though.

[3]Geodesics are the curves that divide the space into two congruent parts, like the straight lines of the Euclidean plane, or the great circles of the sphere. The nexus of geodesics has all the projective properties that define the most basic structure of the Euclidean plane (Coxeter, 1961).

[4]In the worst case the local neighborhood would not even have a projective structure (Coxeter, 1949; note 6). Assume that we are in the Riemannian case, which respects the projective structure. In the general Riemannian case the local neighborhood of a point is like an anisotropically scaled patch of the Euclidean plane (Coxeter, 1961). This implies a periodicity of 180º in Figure 4, but rules out the (actually found) periodicity of 90º.

[5]A possible way to escape the logical conclusion is to widen the scope of potential structures to include “Finsler planes.” In a Finsler plane (Berwald, 1941) even the infinitesimal neighborhoods are unlike the Euclidean plane. We do not consider this arcane model because we fail to see how it might be empirically tested.

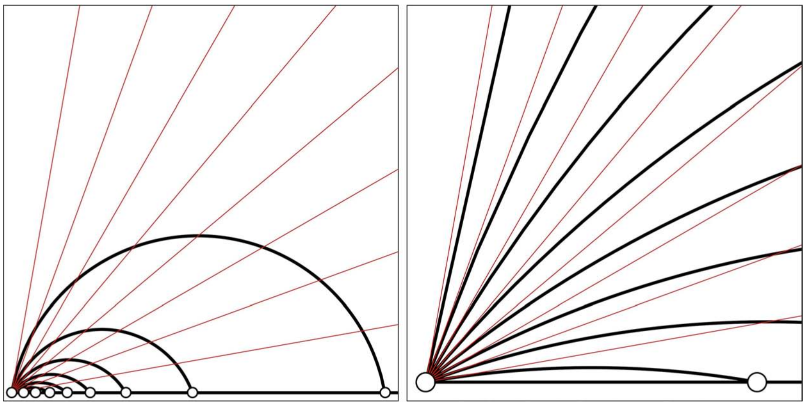

Figure 7. These are predictions for one of the classical simple geometries (Lobachevsky's hyperbolic plane; Coxeter 1965).[6] The fat black lines are geodesics on a Euclidean straight line through the origin. Notice that these geodesics leave the origin in different directions (perhaps better seen in the right figure, which is simply an enlarged part of the plane near the origin). This type of configuration is entirely typical for the classical geometries. It fails to fit our findings, for which all points are pointed to in the same direction.

participants was to point in one of the directions 0º, 45º, 90º, 135º, 180º, 225º, 270º, or 315º. This was repeated 20 times for each of the directions. As it seemed irrelevant, possibly even disadvantageous because of occasional repeats, to use a random order, we simply cycled through the sorted list of directions 20 times. This at least ensures that identical cases are as far apart (in time) as possible.

A summary of the results is shown in Figure 11. The first thing that immediately strikes one when looking at these results is the difference between the settings for the horizontal/vertical and those for the diagonal directions. Indeed, the quantitative difference is so large that the difference is best described as categorical. The participants seem to carry "bubble levels" and "plumb lines" in their eyes, but no "protractors."

For this experiment we ran similar double checks as in the previous one. We can now rule out that the observers used any references that might have been present in the room or the monitor.

Figure 8. The image at right is a representation of the image at left for the case that the "straight lines" through the origin (the center of the image) are transformed according to a directional transformation with periodicity 90º (Figure 4). This fails to be a *linear* transformation, which is an anisotropic scaling. Thus it cannot represent the local effect of any arbitrary smooth mapping. It represents our findings, but it essentially rules out an interpretation in terms of some non-Euclidean geometry.

[6]There are various ways to relate Lobachevsky's plane to the Euclidean plane. For this example we use "Poincaré's half-plane" as a convenient model (Coxeter, 1965). Introducing Cartesian coordinates (x, y), the Euclidean metric is $ds^2 = dx^2 + dy^2$, whereas the metric of the non-Euclidean model is $ds^2 = (dx^2 + dy^2)/y^2$. The half-plane is the part $y > 0$. In our example the origin is taken as $x = 0$, $y = 1$, and the points to be pointed at on the line $y = 1$. The geodesics of the Poincaré's half-plane are (Euclidean) circles with centers on the line $y = 0$ (which is at "infinity" in this metric). This is sufficient to construct the configuration shown in Figure 7.

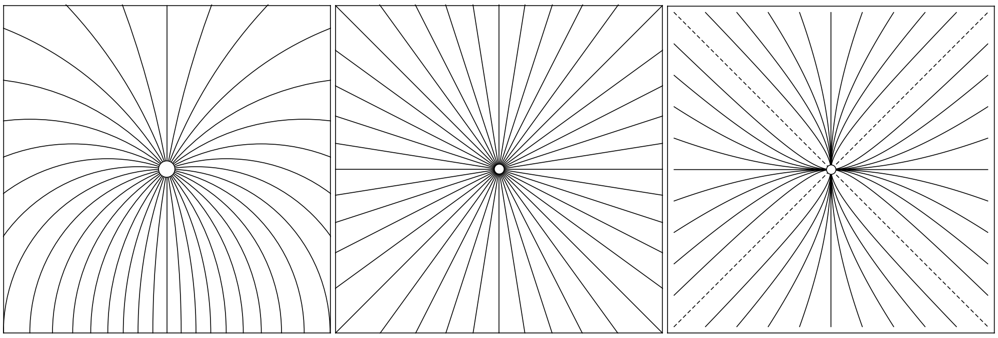

Figure 9. The figure at left shows the case illustrated in Figure 7 (Lobachevsky's hyperbolic plane). The figure at the center shows a highly magnified view of the center. It looks like the simple Euclidean configuration of half-lines fanning out from a point, uniformly spreading over all directions. This occurs for any Riemannian (non-Euclidean) case. The figure at right shows an extreme case of non-Euclidean geometry. Here the geodesics "fanning out" from the point at center all start out in either a horizontal or vertical direction. There is no Riemannian space at all that would look like this. Thus the case at right is worse than a mere arbitrary warping of the Euclidean plane; it is even more "non-Euclidean" than that. Whether one would consider calling it a "plane" at all is a matter of choice.

For the horizontal/vertical directions the effect is mainly a small counter-clockwise overall rotation. It is less than half a degree, in the case of participant JK, hardly significantly different from veridical. We tested whether this trend was due to the sequence order by running a session in the reverse direction. This made no difference.

For the diagonal directions the observers JK and JW show an overall counter-clockwise rotation of several degrees. Participant AD shows a pattern that implies a trend towards the horizontal, of about 10º. The windmill plots (Figure 12) are perhaps more immediately intuitive. (Mind that the gray ranges indicate quartile intervals, *magnified 10 times* about the median!)

If the perceptions of the horizontal and vertical are indeed categorical, then the task of indicating a diagonal direction is perhaps to be interpreted as an angular bisection task. It is well known that such

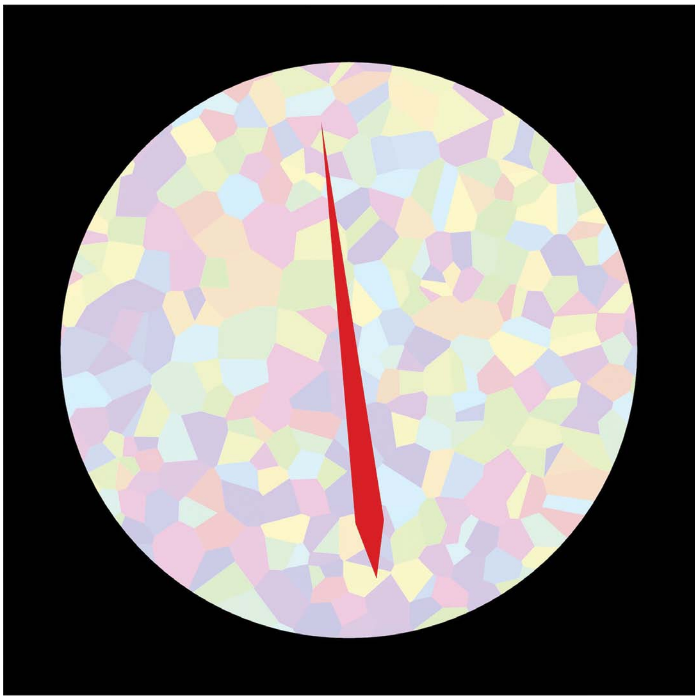

Figure 10. The stimulus array with the large pointer (the red arrow), superimposed on a circular background disk filled with random Voronoi cells of 20% saturation. The texture was refreshed on each trial, in order to prevent the observer from using landmarks. The pointing direction is the direction indicated by the long tapered bi-angle (pointing upwards and slightly to the left in this example). The pointer may be rotated about the center of the background disk.

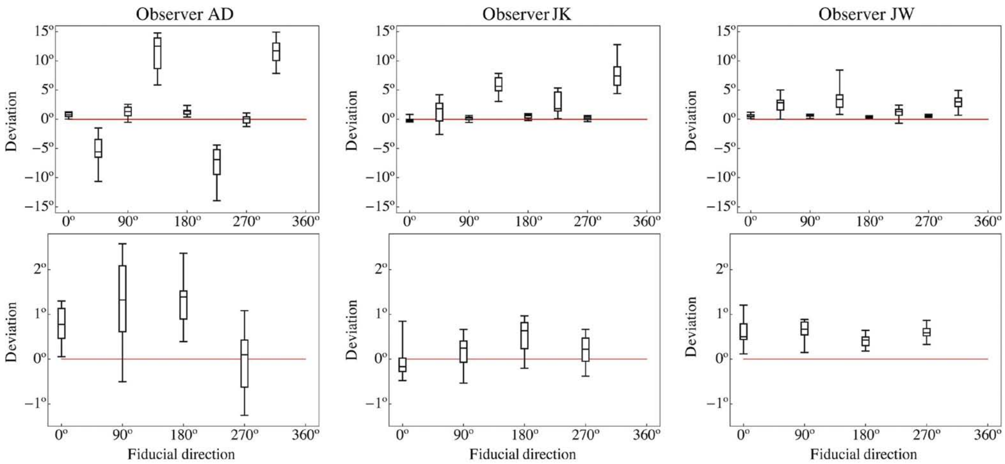

Figure 11. The whisker dot plots show quartiles (the box, with median horizontal line), and 5%, 95% quantiles (the whiskers). The top row shows all data, the bottom row only the horizontal and vertical instances. Notice the difference in scale of the top and bottom rows. Otherwise the scales are identical for all observers, thus enabling comparison of inter-observer differences.

tasks are prone to reveal left–right biases, perhaps due to pseudoneglect (Jewell & McCourt, 2000).[7] This might account for the effect seen in the data of AD.

Judgments of angle size are known to depend upon orientation (MacLean & Stacey, 1971). This might possibly affect the results of bisection tasks, although this would need empirical backing up.

Of course, the fact that diagonal directions are prone to show a variety of "oblique effects" is well known (Meng & Qian, 2005).

It has been speculated that the anisotropic nature of directional judgments finds its origin in the statistics of the natural world (Girshick et al., 2011). This is certainly a possibility to pursue, though this hypothesis does not allow us to predict the specific results of our experiments.

It is somewhat harder to account for the apparent counter-clockwise rotations seen in the data of participants JK and JW. Since the vestibular system is doubtless involved, skull spatial attitudes might have been slightly off (McFarland & Clarkson, 1966; Rock, 1954). Since monocular viewing was used, there may have been effects of ocular torsion (Goonetilleke, Mezey, Burgess, & Curthoys, 2008).

5 Conclusions

We have explored a pure visual field effect that appears to be novel, though no doubt connected to a variety of well-known facts. An "explanation" in terms of neurological, physiological, anatomical, or physical factors is not forthcoming. One can only speculate. Here we remain on the purely phenomenological level, a level that has its own forms of lawfulness (we reserve "causal" to the functional relations between purely physical phenomena). In the analysis of the experiments we mentioned a number of possible connections with fields at ontological levels below psychical phenomenology.

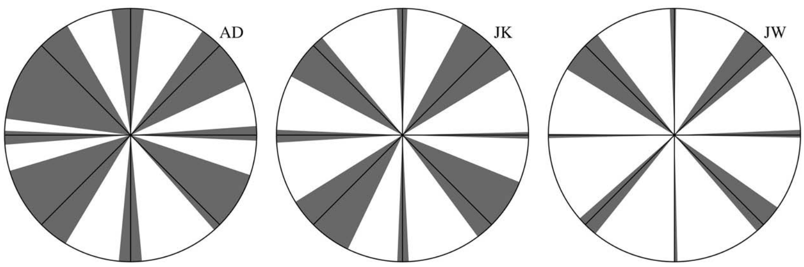

Figure 12. These windmill plots should be read with some care: the gray sectors indicate quartiles ranges, magnified 10 times about the median values. This magnification is a convenient visual aid, because the angular widths of the sectors are really quite small (although significant) in all cases.

[7] *Pseudoneglect:* "A mild asymmetry in spatial attention, displayed by neurologically normal individuals, in which the left side of space tends to be favored, making leftward errors in line bisections as well as in the judgment of brightness, numerosity, and size quite common." (http://en.wiktionary.org/wiki/pseudoneglect)

An interesting general conclusion is that the effects discussed here occur without notable change in both pure visual field settings and in pictorial space. This illustrates once more that pictorial space should not be thought of as "three-dimensional," but rather as "two-plus-one-dimensional," the single dimension being "depth" (Koenderink et al., 2011). We have found this on numerous occasions. Formally, this implies that the geometry is that of a fiber space, with the visual field as base space, and the depth dimension as fibers (Koenderink & van Doorn, 2012). These spaces have fully distinct geometrical structures (the base space approximately Euclidean, the fibers close to affine) and are largely independent of each other. Metrics are of the nature of space-dependent gauge fields; the space may have both curvature and torsion, typically concentrated on singular boundaries of homogeneous regions.

Another interesting (very) general conclusion is that exocentric pointing in the visual field cannot be described through a space of the Riemannian type. The implications of this empirical fact are hard to fathom. One likely possibility is that the pointing task addresses functional connections that are active "on top of" an underlying geometrical structure. Pseudoneglect[7] is a possible contributing factor.

Finally, we mention that the question that led us to embark on this study has been fully resolved. The surprising pattern of systematic mis-pointings in pictorial space is an intrinsic property of the visual field. It has nothing to do with the depth dimension, or pictorial space in general. Moreover, it is not due to the fact that most pictures are bounded by a rectangular frame.

Acknowledgments. This work was supported by the Methusalem program by the Flemish Government (METH/08/02), awarded to Johan Wagemans. We acknowledge administrative support by Agna Marien and technical support by Rudy Dekeerschieter.

References

Berwald, L. (1941). On Finsler and Cartan geometries. III. Two-dimensional spaces with rectilinear extremals. *Annals of Mathematics, 42*(1), 84–112.

Bouma, H., & Andriessen, J. J. (1968). Perceived orientation of isolated line segments. *Vision Research, 8*, 493–507. doi:10.1016/0042-6989(68)90091-6

Coxeter, H. S. M. (1949). *The real projective plane.* New York: McGraw-Hill.

Coxeter, H. S. M. (1961). *Introduction to geometry.* New York: Wiley.

Coxeter. H. S. M. (1965). *Non-Euclidean geometry*. New York: Springer.

Girshick, A. R., Landy, M. S., & Simoncelli, E. P. (2011). Cardinal rules: Visual orientation perception reflects knowledge of environmental statistics. *Nature Neuroscience, 14*, 926–932. doi:10.1038/nn.2831

Goonetilleke, S. C., Mezey, L. E., Burgess, A. M., & Curthoys, I. S. (2008). On the relation between ocular torsion and visual perception of line orientation. *Vision Research, 48*, 1488–1496. doi:10.1016/j.visres.2008.03.012

Helmholtz, H. von (1867). *Handbuch der Physiologischen Optik.* Leipzig: L Voss.

Hess, R. F. (1982). Developmental sensory impairment: Amblyopia or tarachopia? *Human Neurobiology, 1*(1), 17–29.

Jewell, G., & McCourt, M. E. (2000). Pseudoneglect: A review and meta-analysis of performance factors in line bisection tasks. *Neuropsychologia, 38*, 93–110. doi:10.1016/s0028-3932(99)00045-7

Koenderink, J. J., & van Doorn, A. J. (2012). Gauge fields in pictorial space. *SIAM Journal on Imaging Sciences, 5*, 1213–1233.

Koenderink, J. J., van Doorn, A. J., & Lappin, J. S. (2000). Direct measurement of the curvature of visual space. *Perception, 29*, 69–79. doi:10.1068/p2921

Koenderink, J. J., van Doorn, A. J., & Wagemans, J. (2011). Depth. *i-Perception, 2*, 541–562. doi:10.1068/i0438aap

MacLean, I. E., & Stacey, B. G. (1971). Judgment of angle size: An experimental appraisal *Perception & Psychophysics, 9*, 499–504. doi:10.3758/BF03208965

McFarland, J. H., & Clarkson, F. (1966). Perception of orientation: Adaptation to lateral body tilt *American Journal of Psychology, 79*, 265–271.

Meng, X., & Qian, N. (2005). The oblique effect depends on perceived, rather than physical, orientation and direction. *Vision Research, 45*, 3402–3413. doi.org/10.1016/j.visres.2005.05.016

Quinn, P. (2004). Visual perception of orientation is categorical near vertical and continuous near horizontal. *Perception, 33*, 897–906. doi:10.1068/p3453

Rock, I. (1954). The perception of the egocentric orientation of a line. *Journal of Experimental Psychology, 48*, 367–374. doi:10.1037/h0057319

Smythies, J. (1996). A note on the concept of the visual field in neurology, psychology, and visual neuroscience. *Perception, 25*(3), 369–371. doi:10.1068/p250369

Wagemans, J., van Doorn, A. J., & Koenderink, J. J. (2011). Measuring 3D point configurations in pictorial space. *i-Perception, 2*, 77–111. doi:10.1068/i0420
Wenderoth, P., & White, D. (1979). Angle-matching illusions and perceived line orientation. *Perception, 8*, 565–575. doi:10.1068/p080565
Westheimer, G., & Beard, B. L. (1998). Orientation dependency for foveal line stimuli: Detection and intensity discrimination, resolution, orientation discrimination and vernier acuity. *Vision Research, 38*, 1097–1103. doi:10.1016/S0042-6989(97)00248-4

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